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21

Balancing of Rotating Masses

Features

21.1. Introduction

The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations. In this chapter we shall discuss the balancing of unbalanced forces caused by rotating masses, in order to minimise pressure on the main bearings when an engine is running.

21.2. Balancing of Rotating Masses

We have already discussed, that whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a

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way that the centrifugal force of both the masses are made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called ***balancing of rotating masses.***

The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.
2. Balancing of a single rotating mass by two masses rotating in different planes.
3. Balancing of different masses rotating in the same plane.
4. Balancing of different masses rotating in different planes.

We shall now discuss these cases, in detail, in the following pages.

21.3. Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig. 21.1. Let r_1 be the radius of rotation of the mass m_1 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1).

We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1 \qquad \ldots (i)$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite.

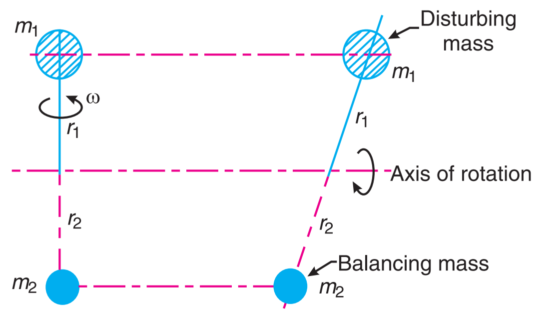


Fig. 21.1. Balancing of a single rotating mass by a single mass rotating in the same plane.

Let r_2 = Radius of rotation of the balancing mass m_2 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

∴ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \qquad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1.\omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Notes : 1. The product $m_2.r_2$ may be split up in any convenient way. But the radius of rotation of the balancing mass (m_2) is generally made large in order to reduce the balancing mass m_2.

2. The centrifugal forces are proportional to the product of the mass and radius of rotation of respective masses, because ω^2 is same for each mass.

21.4. Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

We have discussed in the previous article that by introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced. In other words, the two forces are equal in magnitude and opposite in direction. But this type of arrangement for balancing gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for ***static balancing.***
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero.

The conditions **(1)** and **(2)** together give ***dynamic balancing.*** The following two possibilities may arise while attaching the two balancing masses :

1. The plane of the disturbing mass may be in between the planes of the two balancing masses, and
2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

We shall now discuss both the above cases one by one.

The picture shows a diesel engine. All diesel, petrol and steam engines have reciprocating and rotating masses inside them which need to be balanced.

1. ***When the plane of the disturbing mass lies in between the planes of the two balancing masses***

Consider a disturbing mass m lying in a plane A to be balanced by two rotating masses m_1 and m_2 lying in two different planes L and M as shown in Fig. 21.2. Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, L and M respectively.

Let l_1 = Distance between the planes A and L,

l_2 = Distance between the planes A and M, and

l = Distance between the planes L and M.

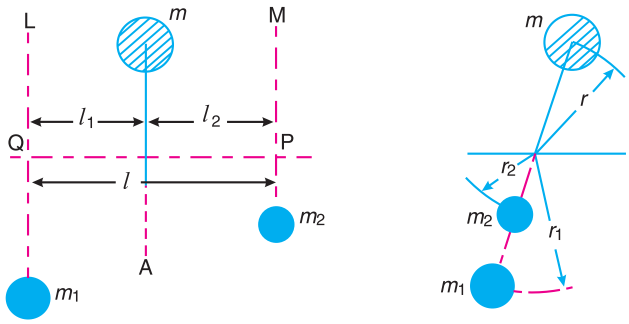


Fig. 21.2. Balancing of a single rotating mass by two rotating masses in different planes when the plane of single rotating mass lies in between the planes of two balancing masses.

We know that the centrifugal force exerted by the mass m in the plane A,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane M,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \quad \text{or} \quad m \cdot \omega^2 \cdot r = m_1.\omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\therefore \quad m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \quad \quad \text{. . . (i)}$$

Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \quad \quad \text{. . . (ii)}$$

Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$\therefore \quad m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \quad \quad \text{. . . (iii)}$$

It may be noted that equation ***(i)*** represents the condition for static balance, but in order to achieve dynamic balance, equations ***(ii)*** or ***(iii)*** must also be satisfied.

2. *When the plane of the disturbing mass lies on one end of the planes of the balancing masses*

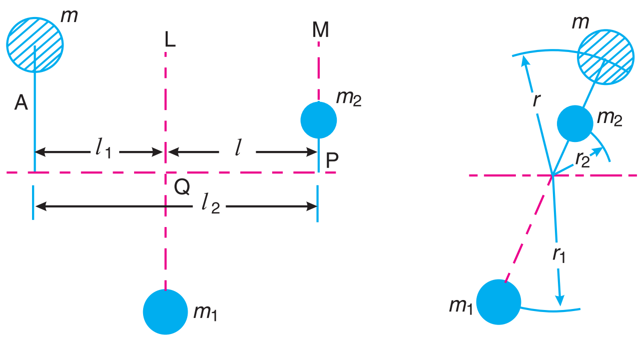


Fig. 21.3. Balancing of a single rotating mass by two rotating masses in different planes, when the plane of single rotating mass lies at one end of the planes of balancing masses.

In this case, the mass m lies in the plane A and the balancing masses lie in the planes L and M, as shown in Fig. 21.3. As discussed above, the following conditions must be satisfied in order to balance the system, *i.e.*

$$F_C + F_{C2} = F_{C1} \quad \text{or} \quad m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

$$\therefore \quad m \cdot r + m_2 \cdot r_2 = m_1 . r_1 \qquad \text{. . . } (iv)$$

Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \qquad \text{. . . } (v)$$

. . . [Same as equation ***(ii)***]

Similarly, to find the balancing force in the plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

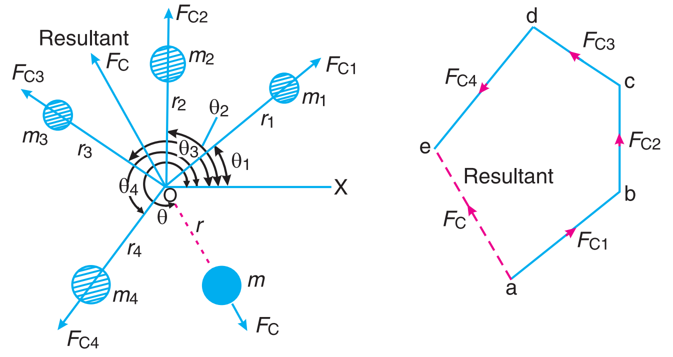
$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \qquad \text{. . . } (vi)$$

. . . [Same as equation ***(iii)***]

21.5. Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let $\theta_1, \theta_2, \theta_3$ and θ_4 be the angles of these masses with the horizontal line OX, as shown in Fig. 21.4 (*a*). Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below :



(*a*) Space diagram. (*b*) Vector diagram.

Fig. 21.4. Balancing of several masses rotating in the same plane.

1. *Analytical method*

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :

1. First of all, find out the centrifugal force* (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.

A car assembly line.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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* Since ω^2 is same for each mass, therefore the magnitude of the centrifugal force for each mass is proportional to the product of the respective mass and its radius of rotation.

2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e.* ΣH and ΣV. We know that

   Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in ***opposite direction.***
6. Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

where m = Balancing mass, and

r = Its radius of rotation.

2. *Graphical method*

The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig. 21.4 (*a*).
2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$).
4. Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig. 21.4 (*b*).
5. The balancing force is, then, equal to the resultant force, but in ***opposite direction.***
6. Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$

or $m.r$ = Resultant of $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$

Example 21.1. *Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.*

Solution. Given : m_1 = 200 kg ; m_2 = 300 kg ; m_3 = 240 kg ; m_4 = 260 kg ; r_1 = 0.2 m ; r_2 = 0.15 m ; r_3 = 0.25 m ; r_4 = 0.3 m ; θ_1 = 0° ; θ_2 = 45° ; θ_3 = 45° + 75° = 120° ; θ_4 = 45° + 75° + 135° = 255° ; r = 0.2 m

Let m = Balancing mass, and

θ = The angle which the balancing mass makes with m_1.

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore

$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$

$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$

$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

The problem may, now, be solved either analytically or graphically. But we shall solve the problem by both the methods one by one.

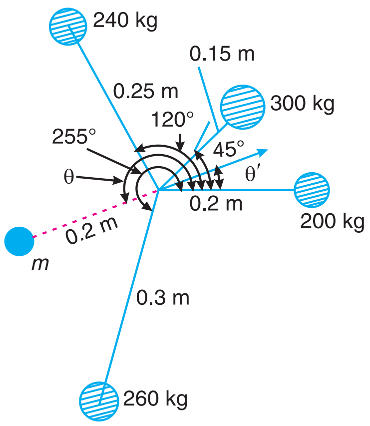


Fig. 21.5

1. *Analytical method*

The space diagram is shown in Fig. 21.5.

Resolving $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ horizontally,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + m_3 \cdot r_3 \cos\theta_3 + m_4 \cdot r_4 \cos\theta_4$$

$$= 40 \cos 0° + 45 \cos 45° + 60 \cos 120° + 78 \cos 255°$$

$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m}$$

Now resolving vertically,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + m_3 \cdot r_3 \sin\theta_3 + m_4 \cdot r_4 \sin\theta_4$$

$$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255°$$

$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m}$$

$\therefore$ Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

We know that

$m \cdot r = R = 23.2$ or $m = 23.2 / r = 23.2 / 0.2 = 116$ kg **Ans.**

and $\tan\theta' = \Sigma V / \Sigma H = 8.5 / 21.6 = 0.3935$ or $\theta' = 21.48°$

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

θ = 180° + 21.48° = 201.48° **Ans.**

2. *Graphical method*

The magnitude and the position of the balancing mass may also be found graphically as discussed below :

1. First of all, draw the space diagram showing the positions of all the given masses as shown in Fig 21.6 (*a*).
2. Since the centrifugal force of each mass is proportional to the product of the mass and radius, therefore

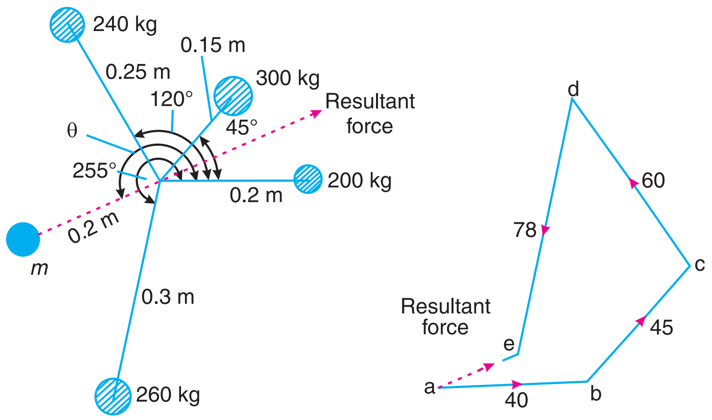
$m_1.r_1$ = 200 × 0.2 = 40 kg-m

$m_2.r_2$ = 300 × 0.15 = 45 kg-m

$m_3.r_3 = 240 \times 0.25 = 60$ kg-m

$m_4.r_4 = 260 \times 0.3 = 78$ kg-m

3. Now draw the vector diagram with the above values, to some suitable scale, as shown in Fig. 21.6 (*b*). The closing side of the polygon *ae* represents the resultant force. By measurement, we find that *ae* = 23 kg-m.



(*a*) Space diagram. (*b*) Vector diagram

Fig. 21.6

4. The balancing force is equal to the resultant force, but ***opposite*** in direction as shown in Fig. 21.6 (*a*). Since the balancing force is proportional to *m.r*, therefore

$m \times 0.2$ = vector ea = 23 kg-m or $m = 23/0.2 =$ **115 kg Ans.**

By measurement we also find that the angle of inclination of the balancing mass (*m*) from the horizontal mass of 200 kg,

$\theta = 201°$ **Ans.**

21.6. Balancing of Several Masses Rotating in Different Planes

When several masses revolve in different planes, they may be transferred to a ***reference plane*** (briefly written as ***R.P.***), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero.

2. The couples about the reference plane must balance, *i.e.* the resultant couple must be zero.

Diesel engine.

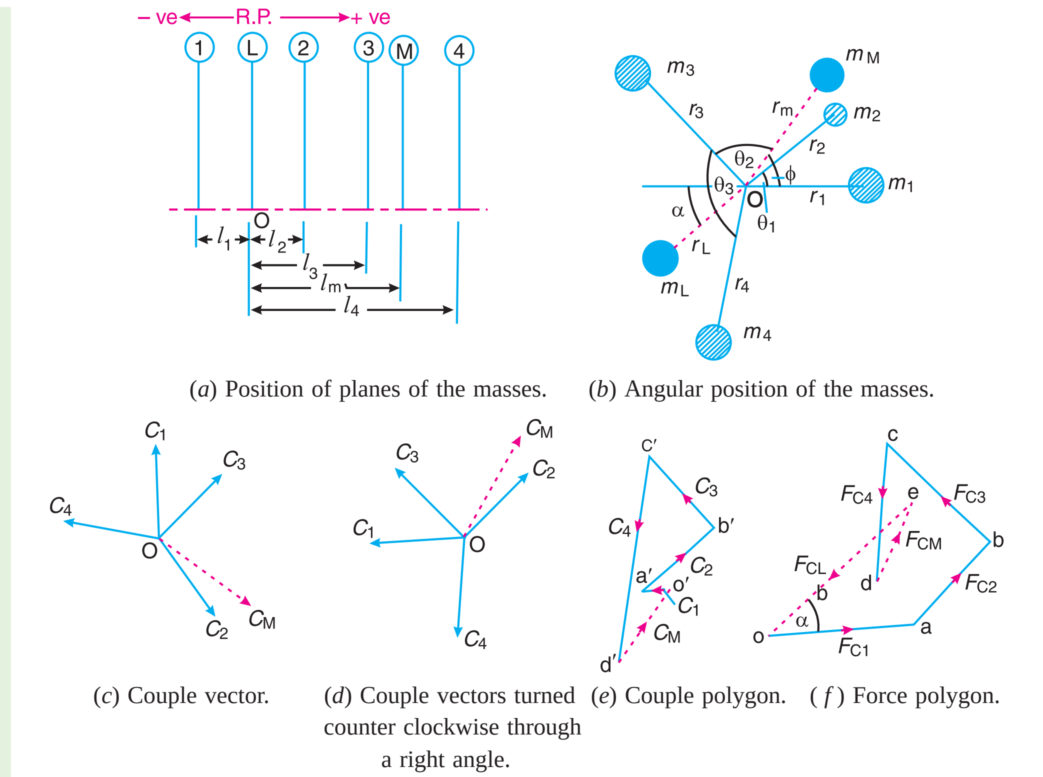
Let us now consider four masses m_1, m_2, m_3 and m_4 revolving in planes 1, 2, 3 and 4 respectively as shown in

Fig. 21.7 (*a*). The relative angular positions of these masses are shown in the end view [Fig. 21.7 (*b*)]. The magnitude of the balancing masses m_L and m_M in planes *L* and *M* may be obtained as discussed below :

1. Take one of the planes, say *L* as the reference plane (*R.P.*). The distances of all the other planes to the left of the reference plane may be regarded as ***negative,*** and those to the right as ***positive.***
2. Tabulate the data as shown in Table 21.1. The planes are tabulated in the same order in which they occur, reading from left to right.

Table 21.1

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(*R.P.*)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



(*a*) Position of planes of the masses. (*b*) Angular position of the masses.

(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

Fig. 21.7. Balancing of several masses rotating in different planes.

3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring m_1 to the reference plane through *O* is propor-

tional to $m_1.r_1.l_1$ and acts in a plane through Om_1 and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to Om_1 as shown by OC_1 in Fig. 21.7 (*c*). Similarly, the vectors OC_2, OC_3 and OC_4 are drawn perpendicular to Om_2, Om_3 and Om_4 respectively and in the plane of the paper.

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig. 21.7 (*d*). We see that their relative positions remains unaffected. Now the vectors OC_2, OC_3 and OC_4 are parallel and in the same direction as Om_2, Om_3 and Om_4, while the vector OC_1 is parallel to Om_1 but in *opposite direction. Hence the ***couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.***

5. Now draw the couple polygon as shown in Fig. 21.7 (*e*). The vector $d'o'$ represents the balanced couple. Since the balanced couple C_M is proportional to $m_M.r_M.l_M$, therefore

$$C_M = m_M \cdot r_M \cdot l_M = \text{vector } d'o' \quad \text{or} \quad m_M = \frac{\text{vector } d'o'}{r_M \cdot l_M}$$

From this expression, the value of the balancing mass m_M in the plane *M* may be obtained, and the angle of inclination ϕ of this mass may be measured from Fig. 21.7 (*b*).

6. Now draw the force polygon as shown in Fig. 21.7 (*f*). The vector *eo* (in the direction from *e* to *o*) represents the balanced force. Since the balanced force is proportional to $m_L.r_L$, therefore,

$$m_L \cdot r_L = \text{vector } eo \quad \text{or} \quad m_L = \frac{\text{vector } eo}{r_L}$$

From this expression, the value of the balancing mass m_L in the plane *L* may be obtained and the angle of inclination α of this mass with the horizontal may be measured from Fig. 21.7 (*b*).

Example 21.2. *A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.*

Solution. Given : m_A = 200 kg ; m_B = 300 kg ; m_C = 400 kg ; m_D = 200 kg ; r_A = 80 mm = 0.08m ; r_B = 70 mm = 0.07 m ; r_C = 60 mm = 0.06 m ; r_D = 80 mm = 0.08 m ; $r_X = r_Y$ = 100 mm = 0.1 m

Let m_X = Balancing mass placed in plane *X*, and

m_Y = Balancing mass placed in plane *Y*.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 21.8 (*a*) and (*b*) respectively.

Assume the plane *X* as the reference plane (*R.P.*). The distances of the planes to the right of plane *X* are taken as + ve while the distances of the planes to the left of plane *X* are taken as – ve. The data may be tabulated as shown in Table 21.2.

* From Table 21.1 (column 6) we see that the couple is – $m_1.r_1.l_1$.

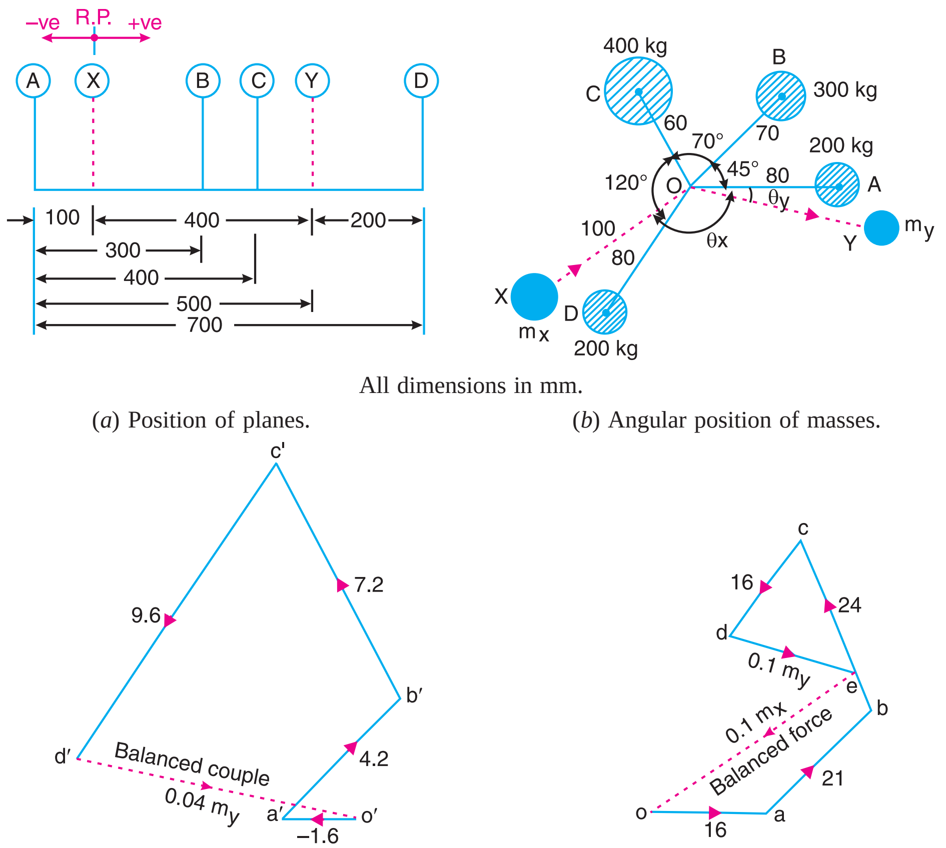
Table 21.2

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent.force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from Plane x(l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

The balancing masses m_X and m_Y and their angular positions may be determined graphically as discussed below :

1. First of all, draw the couple polygon from the data given in Table 21.2 (column 6) as shown in Fig. 21.8 (*c*) to some suitable scale. The vector $d'o'$ represents the balanced couple. Since the balanced couple is proportional to 0.04 m_Y, therefore by measurement,

$$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2 \quad \text{or} \quad m_Y = 182.5 \text{ kg} \quad \textbf{Ans.}$$



All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 21.8

The angular position of the mass m_Y is obtained by drawing Om_Y in Fig. 21.8 (*b*), parallel to vector $d'o'$. By measurement, the angular position of m_Y is $\theta_Y = 12°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

2. Now draw the force polygon from the data given in Table 21.2 (column 4) as shown in Fig. 21.8 (*d*). The vector *eo* represents the balanced force. Since the balanced force is proportional to 0.1 m_X, therefore by measurement,

$$0.1\, m_X = \text{vector } eo = 35.5 \text{ kg-m} \quad \text{or} \quad m_X = 355 \text{ kg } \textbf{Ans.}$$

The angular position of the mass m_X is obtained by drawing Om_X in Fig. 21.8 (*b*), parallel to vector *eo*. By measurement, the angular position of m_X is $\theta_X = 145°$ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

Example 21.3. *Four masses A, B, C and D as shown below are to be completely balanced.*

	A	*B*	*C*	*D*
Mass (kg)	—	*30*	*50*	*40*
Radius (mm)	*180*	*240*	*120*	*150*

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

1. *The magnitude and the angular position of mass A ; and*

2. *The position of planes A and D.*

Solution. Given : r_A = 180 mm = 0.18 m ; m_B = 30 kg ; r_B = 240 mm = 0.24 m ; m_C = 50 kg ; r_C = 120 mm = 0.12 m ; m_D = 40 kg ; r_D = 150 mm = 0.15 m ; $\angle BOC$ = 90° ; $\angle BOD$ = 210° ; $\angle COD$ = 120°

1. *The magnitude and the angular position of mass A*

Let m_A = Magnitude of Mass *A*,

x = Distance between the planes *B* and *D*, and

y = Distance between the planes *A* and *B*.

The position of the planes and the angular position of the masses is shown in Fig. 21.9 (*a*) and (*b*) respectively.

Assuming the plane *B* as the reference plane (*R.P.*) and the mass *B* (m_B) along the horizontal line as shown in Fig. 21.9 (*b*), the data may be tabulated as below :

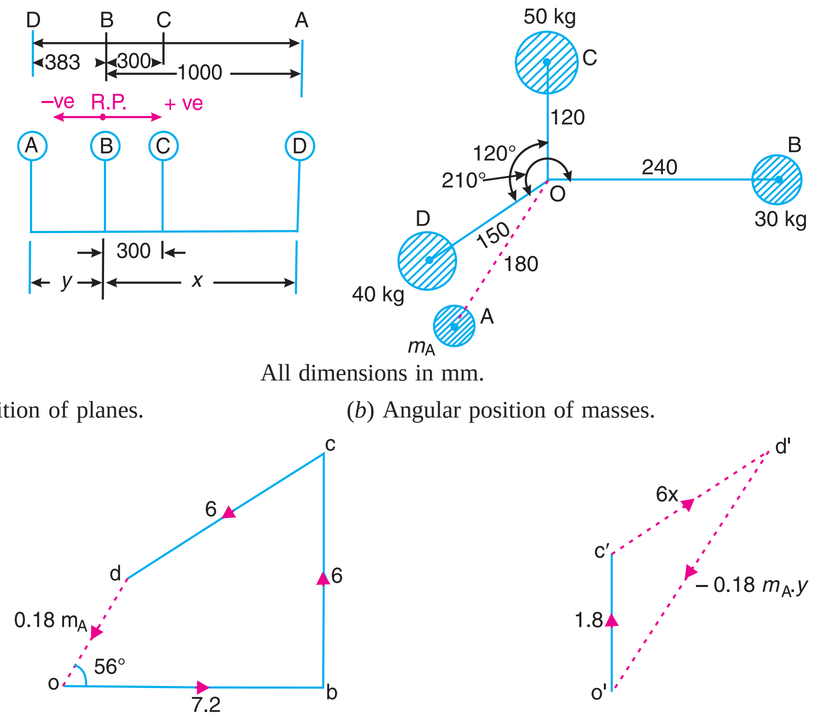
Table 21.3

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent.force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane B (l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* (6)
A	m_A	0.18	0.08 m_A	$-y$	$-0.18\, m_A.y$
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	6x

The magnitude and angular position of mass *A* may be determined by drawing the force polygon from the data given in Table 21.3 (Column 4), as shown in Fig. 21.9 (*c*), to some suitable scale. Since the masses are to be completely balanced, therefore the force polygon must be a closed figure. The closing side (*i.e.* vector *do*) is proportional to 0.18 m_A. By measurement,

$$0.18\, m_A = \text{Vector } do = 3.6 \text{ kg-m} \quad \text{or} \quad m_A = 20 \text{ kg} \quad \textbf{Ans.}$$

In order to find the angular position of mass *A*, draw *OA* in Fig. 21.9 (*b*) parallel to vector *do*. By measurement, we find that the angular position of mass *A* from mass *B* in the anticlockwise direction is $\angle AOB$ = **236° Ans.**



All dimensions in mm.

(*a*) Position of planes.

(*b*) Angular position of masses.

(*c*) Force polygon.

(*d*) Couple polygon.

Fig. 21.9.

2. *Position of planes A and D*

The position of planes *A* and *D* may be obtained by drawing the couple polygon, as shown in Fig. 21.9 (*d*), from the data given in Table 21.3 (column 6). The couple polygon is drawn as discussed below :

1. Draw vector $o'c'$ parallel to *OC* and equal to 1.8 kg-m^2, to some suitable scale.

2. From points c' and o', draw lines parallel to *OD* and *OA* respectively, such that they intersect at point d'. By measurement, we find that

$$6\,x = \text{vector } c'd' = 2.3 \text{ kg-m}^2 \text{ or } x = 0.383 \text{ m}$$

We see from the couple polygon that the direction of vector $c'd'$ is opposite to the direction of mass *D*. Therefore the plane of mass *D* is 0.383 m or 383 mm towards left of plane *B* and not towards right of plane *B* as already assumed. **Ans.**

Again by measurement from couple polygon,

$$-0.18\, m_A.y = \text{vector } o'd' = 3.6 \text{ kg-m}^2$$

$$-0.18 \times 20\, y = 3.6 \quad \text{or} \quad y = -1 \text{ m}$$

The negative sign indicates that the plane *A* is not towards left of *B* as assumed but it is 1 m or 1000 mm towards right of plane *B*. **Ans.**

Example 21.4. *A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively.*

Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

Solution. Given : r_A = 100 mm = 0.1 m ; r_B = 125 mm = 0.125 m ; r_C = 200 mm = 0.2 m ; r_D = 150 mm = 0.15 m ; m_B = 10 kg ; m_C = 5 kg ; m_D = 4 kg

The position of planes is shown in Fig. 21.10 (*a*). Assuming the plane of mass *A* as the reference plane (*R.P.*), the data may be tabulated as below :

Table 21.4

Plane (1)	*Mass (m)* *kg* (2)	*Radius (r)* *m* (3)	*Cent. Force* $\div \omega^2$ *(m.r)kg-m* (4)	*Distance from plane A (l)m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

First of all, the angular setting of masses *C* and *D* is obtained by drawing the couple polygon from the data given in Table 21.4 (column 6). Assume the position of mass *B* in the horizontal direction *OB* as shown in Fig. 21.10 (*b*). Now the couple polygon as shown in Fig. 21.10 (*c*) is drawn as discussed below :

1. Draw vector *o′ b′* in the horizontal direction (*i.e.* parallel to *OB*) and equal to 0.75 kg-m^2, to some suitable scale.

2. From points *o′* and *b′*, draw vectors *o′ c′* and *b′ c′* equal to 1.2 kg-m^2 and 1.08 kg-m^2 respectively. These vectors intersect at *c′*.

3. Now in Fig. 21.10 (*b*), draw *OC* parallel to vector *o′ c′* and *OD* parallel to vector *b′ c′*.

By measurement, we find that the angular setting of mass *C* from mass *B* in the anticlockwise direction, *i.e.*

$$\angle BOC = 240° \quad \textbf{Ans.}$$

and angular setting of mass *D* from mass *B* in the anticlockwise direction, *i.e.*

$$\angle BOD = 100° \quad \textbf{Ans.}$$

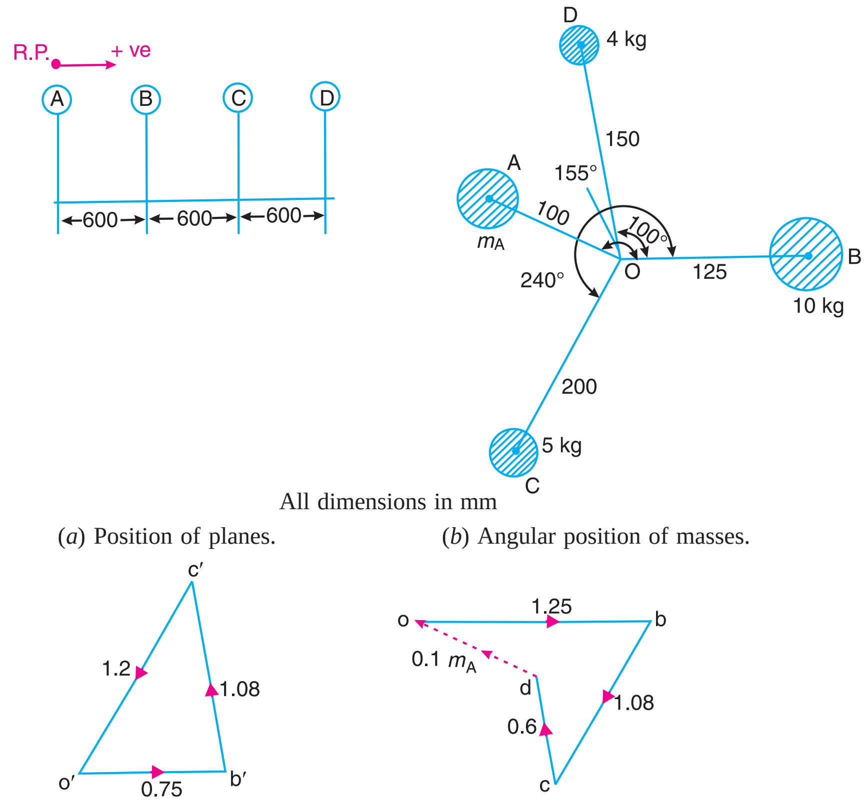
In order to find the required mass *A* (m_A) and its angular setting, draw the force polygon to some suitable scale, as shown in Fig. 21.10 (*d*), from the data given in Table 21.4 (column 4).

Since the closing side of the force polygon (vector *do*) is proportional to 0.1 m_A, therefore by measurement,

$$0.1\, m_A = 0.7 \text{ kg-m}^2 \quad \text{or} \quad m_A = 7 \text{ kg} \quad \textbf{Ans.}$$

Now draw OA in Fig. 21.10 (b), parallel to vector do. By measurement, we find that the angular setting of mass A from mass B in the anticlockwise direction, *i.e.*

$\angle BOA = 155°$ **Ans.**



All dimensions in mm

(a) Position of planes. (b) Angular position of masses.

(c) Couple polygon. (d) Force polygon.

Fig. 21.10

Example 21.5. *A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine :*

1. *The magnitude of the masses at A and D ;* ***2.*** *the distance between planes A and D ; and* ***3.*** *the angular position of the mass at D.*

Solution. Given : $m_B = 18$ kg ; $m_C = 12.5$ kg ; $r_B = r_C = 60$ mm $= 0.06$ m ; $r_A = r_D = 80$ mm $= 0.08$ m ; $\angle BOC = 100°$; $\angle BOA = 190°$

1. Magnitude of the masses at A and D

Let M_A = Mass at A,

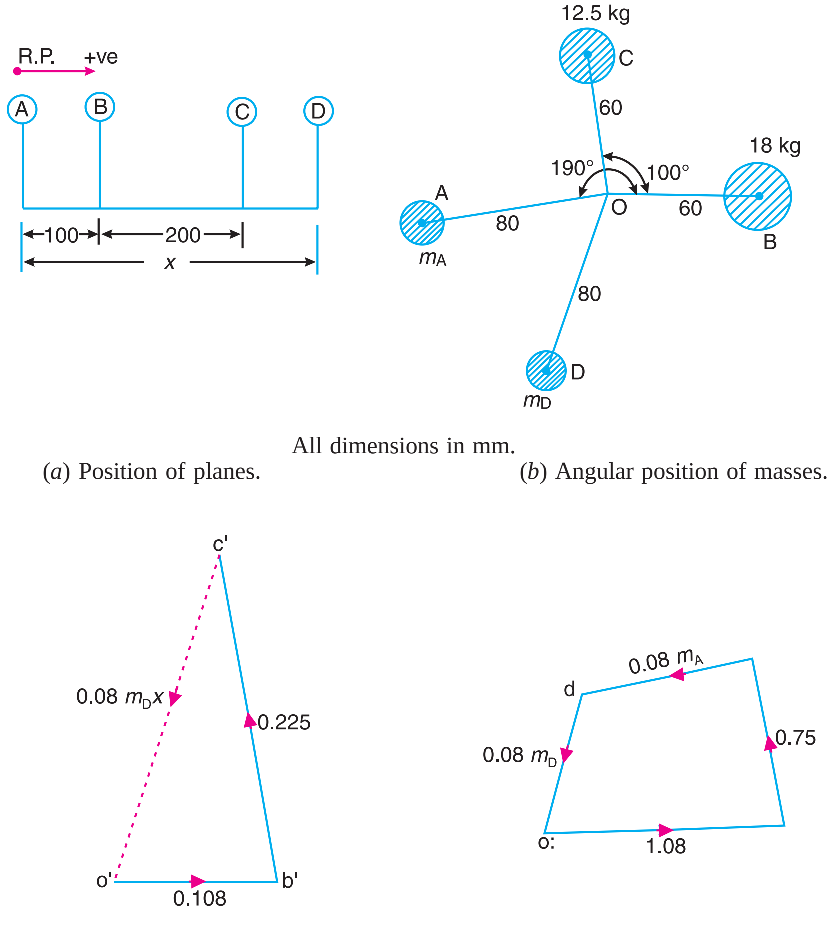
M_D = Mass at D, and

x = Distance between planes A and D.

The position of the planes and angular position of the masses is shown in Fig. 21.11 (*a*) and (*b*) respectively. The position of mass *B* is assumed in the horizontal direction, *i.e.* along *OB*. Taking the plane of mass *A* as the reference plane, the data may be tabulated as below :

Table 21.5

Plane (1)	*Mass* (*m*) kg (2)	*Eccentricity* (*r*) m (3)	*Cent. force* $\div \omega^2$ (*m.r*) kg-m (4)	*Distance from plane A*(*l*)m (5)	*Couple* $\div \omega^2$ (*m.r.l*) kg-m^2 (6)
A (*R.P.*)	m_A	0.08	0.08 m_A	0	0
B	18	0.06	1.08	0.1	0.108
C	12.5	0.06	0.75	0.3	0.225
D	m_D	0.08	0.08 m_D	*x*	0.08 m_D . *x*



All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.

(c) Couple polygon. (*d*) Force polygon.

Fig. 21.11

First of all, the direction of mass *D* is fixed by drawing the couple polygon to some suitable scale, as shown in Fig. 21.11 (*c*), from the data given in Table 21.5 (column 6). The closing side of the couple polygon (vector $c'o'$) is proportional to 0.08 $m_D.x$. By measurement, we find that

$$0.08\ m_D.x = \text{vector}\ c'o' = 0.235\ \text{kg-m}^2 \quad \dots (i)$$

In Fig. 21.11 (*b*), draw *OD* parallel to vector $c'o'$ to fix the direction of mass *D*.

Now draw the force polygon, to some suitable scale, as shown in Fig. 21.11 (*d*), from the data given in Table 21.5 (column 4), as discussed below :

1. Draw vector *ob* parallel to *OB* and equal to 1.08 kg-m.
2. From point *b*, draw vector *bc* parallel to *OC* and equal to 0.75 kg-m.
3. For the shaft to be in complete dynamic balance, the force polygon must be a closed figure. Therefore from point *c*, draw vector *cd* parallel to *OA* and from point *o* draw vector *od* parallel to *OD*. The vectors *cd* and *od* intersect at *d*. Since the vector *cd* is proportional to 0.08 m_A, therefore by measurement

$$0.08\ m_A = \text{vector}\ cd = 0.77\ \text{kg-m} \quad \text{or} \quad m_A = 9.625\ \text{kg}\ \textbf{Ans.}$$

and vector *do* is proportional to 0.08 m_D, therefore by measurement,

$$0.08\ m_D = \text{vector}\ do = 0.65\ \text{kg-m} \quad \text{or} \quad m_D = 8.125\ \text{kg}\ \textbf{Ans.}$$

2. Distance between planes A and D

From equation **(*i*)**,

$$0.08\ m_D.x = 0.235\ \text{kg-m}^2$$

$$0.08 \times 8.125 \times x = 0.235\ \text{kg-m}^2 \quad \text{or} \quad 0.65\ x = 0.235$$

$$\therefore \quad x = \frac{0.235}{0.65} = 0.3615\text{m} = 361.5\ \text{mm}\ \textbf{Ans.}$$

3. Angular position of mass at D

By measurement from Fig. 21.11 (*b*), we find that the angular position of mass at *D* from mass *B* in the anticlockwise direction, *i.e.* $\angle BOD = 251°$ **Ans.**

Example 21.6. *A shaft has three eccentrics, each 75 mm diameter and 25 mm thick, machined in one piece with the shaft. The central planes of the eccentric are 60 mm apart. The distance of the centres from the axis of rotation are 12 mm, 18 mm and 12 mm and their angular positions are 120° apart. The density of metal is 7000 kg/m³. Find the amount of out-of-balance force and couple at 600 r.p.m. If the shaft is balanced by adding two masses at a radius 75 mm and at distances of 100 mm from the central plane of the middle eccentric, find the amount of the masses and their angular positions.*

Solution. Given : D = 75 mm = 0.075 m ; t = 25 mm = 0.025 m ; r_A = 12 mm = 0.012 m ; r_B = 18 mm = 0.018 m ; r_C = 12 mm = 0.012 mm ; ρ = 7000 kg/m³ ; N = 600 r.p.m. or $\omega = 2\pi \times 600/60$ = 62.84 rad/s ; $r_L = r_M$ = 75 mm = 0.075 m

We know that mass of each eccentric,

$$m_A = m_B = m_C = \text{Volume} \times \text{Density} = \frac{\pi}{4} \times D^2 \times t \times \rho$$

$$= \frac{\pi}{4}(0.075)^2(0.025)7000 = 0.77\ \text{kg}$$

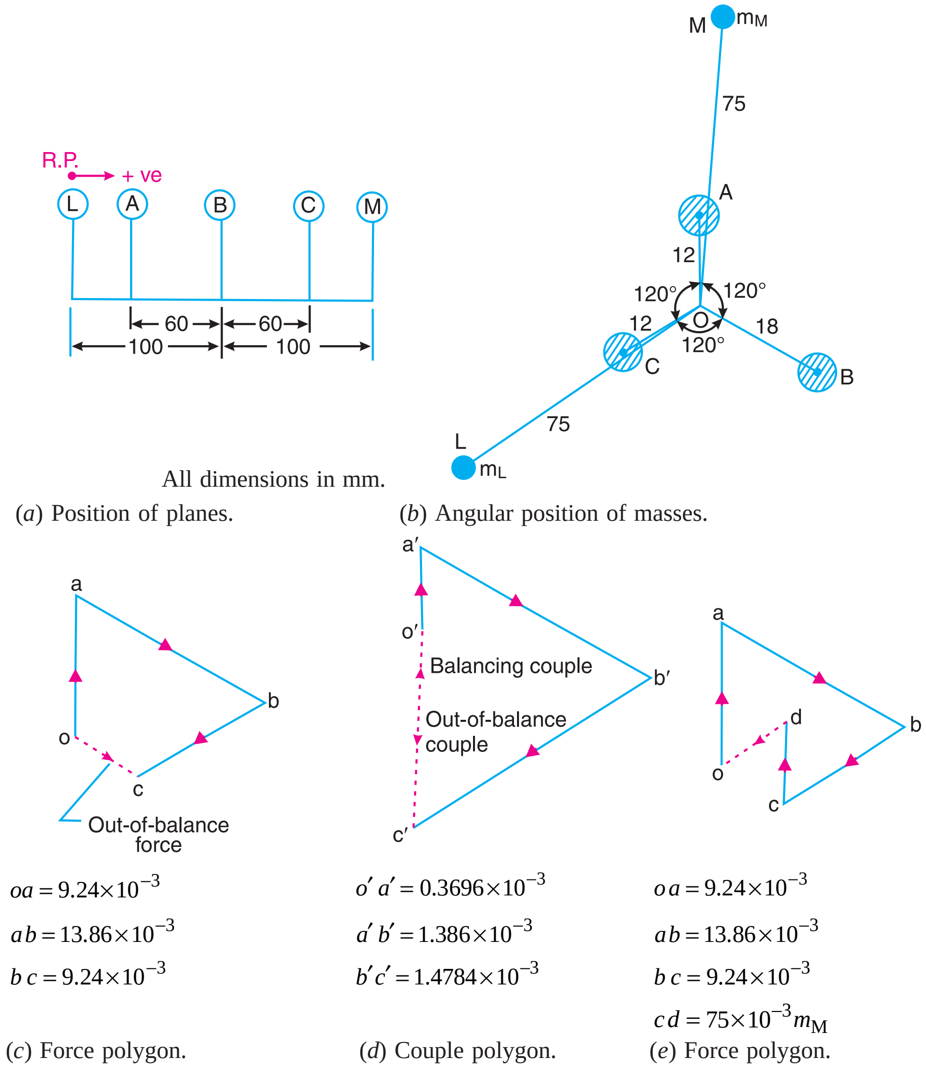
Let *L* and *M* be the planes at distances of 100 mm from the central plane of middle eccentric. The position of the planes and the angular position of the three eccentrics is shown in Fig. 21.12 (*a*) and (*b*) respectively. Assuming *L* as the reference plane and mass of the eccentric *A* in the vertical direction, the data may be tabulated as below :

Table 21.6.

Plane (1)	*Mass* *(m) kg* (2)	*Radius* *(r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane L.(l)m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* (6)
L (*R.P.*)	m_L	0.075	$75 \times 10^{-3}\ m_L$	0	0
A	0.77	0.012	9.24×10^{-3}	0.04	0.3696×10^{-3}
B	0.77	0.018	13.86×10^{-3}	0.1	1.386×10^{-3}
C	0.77	0.012	9.24×10^{-3}	0.16	1.4784×10^{-3}
M	m_M	0.075	$75 \times 10^{-3}\ m_M$	0.20	$15 \times 10^{-3}\ m_M$

Out-of-balance force

The out-of-balance force is obtained by drawing the force polygon, as shown in Fig. 21.12 (*c*), from the data given in Table 21.6 (column 4). The resultant *oc* represents the out-of-balance force.



(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Force polygon. (*d*) Couple polygon. (*e*) Force polygon.

Fig. 21.12

Since the centrifugal force is proportional to the product of mass and radius (*i.e.* $m.r$), therefore by measurement.

Out-of-balance force = vector oc = 4.75×10^{-3} kg-m

$$= 4.75 \times 10^{-3} \times \omega^2 = 4.75 \times 10^{-3} (62.84)^2 = 18.76 \text{ N}$$ **Ans.**

Out-of-balance couple

The out-of-balance couple is obtained by drawing the couple polygon from the data given in Table 21.6 (column 6), as shown in Fig. 21.12 (*d*). The resultant $o'c'$ represents the out-of-balance couple. Since the couple is proportional to the product of force and distance ($m.r.l$), therefore by measurement,

Out-of-balance couple = vector $o'c'$ = 1.1×10^{-3} kg-m^2

$$= 1.1 \times 10^{-3} \times \omega^2 = 1.1 \times 10^{-3} (62.84)^2 = 4.34 \text{ N-m}$$ **Ans.**

Amount of balancing masses and their angular positions

The vector $c'o'$ (in the direction from c' to o'), as shown in Fig. 21.12 (*d*) represents the balancing couple and is proportional to $15 \times 10^{-3}\ m_M$, *i.e.*

$$15 \times 10^{-3}\ m_M = \text{vector } c'o' = 1.1 \times 10^{-3} \text{ kg-m}^2$$

or $$m_M = 0.073 \text{ kg}$$ **Ans.**

Draw OM in Fig. 21.12 (*b*) parallel to vector $c'o'$. By measurement, we find that the angular position of balancing mass (m_M) is 5° from mass A in the clockwise direction. **Ans.**

Ship powered by a diesel engine.

In order to find the balancing mass (m_L), a force polygon as shown in Fig. 21.12 (*e*) is drawn. The closing side of the polygon *i.e.* vector *do* (in the direction from *d* to *o*) represents the balancing force and is proportional to 75 × 10^{-3} m_L. By measurement, we find that

$$75 \times 10^{-3}\, m_L = \text{vector } do = 5.2 \times 10^{-3} \text{ kg-m}$$

or $m_L = 0.0693$ kg **Ans.**

Draw *OL* in Fig. 21.12 (*b*), parallel to vector *do*. By measurement, we find that the angular position of mass (m_L) is 124° from mass *A* in the clockwise direction. **Ans.**

Example 21.7. *A shaft is supported in bearings 1.8 m apart and projects 0.45 m beyond bearings at each end. The shaft carries three pulleys one at each end and one at the middle of its length. The mass of end pulleys is 48 kg and 20 kg and their centre of gravity are 15 mm and 12.5 mm respectively from the shaft axis. The centre pulley has a mass of 56 kg and its centre of gravity is 15 mm from the shaft axis. If the pulleys are arranged so as to give static balance, determine :* **1.** *relative angular positions of the pulleys, and* **2.** *dynamic forces produced on the bearings when the shaft rotates at 300 r.p.m.*

Solution. Given : m_A = 48 kg ; m_C = 20 kg ; r_A = 15 mm = 0.015 m ; r_C = 12.5 mm = 0.0125 m ; m_B = 56 kg ; r_B = 15 mm = 0.015 m ; *N* = 300 *r.p.m.* or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

1. *Relative angular position of the pulleys*

The position of the shaft and pulleys is shown in Fig. 21.13 (*a*).

Let m_L and m_M = Mass at the bearings *L* and *M*, and

r_L and r_M = Radius of rotation of the masses at *L* and *M* respectively.

Assuming the plane of bearing *L* as reference plane, the data may be tabulated as below :

Table 21.7.

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force ÷ ω² (m.r) kg-m* (4)	*Distance from plane L(l)m* (5)	*Couple ÷ ω² (m.r.l) kg-m²* (6)
A	48	0.015	0.72	– 0.45	– 0.324
L(*R.P*)	m_L	r_L	$m_L.r_L$	0	0
B	56	0.015	0.84	0.9	0.756
M	m_M	r_M	$m_M.r_M$	1.8	1.8 $m_M.r_M$
C	20	0.0125	0.25	2.25	0.5625

First of all, draw the force polygon to some suitable scale, as shown in Fig. 21.13 (*c*), from the data given in Table 21.7 (column 4). It is assumed that the mass of pulley *B* acts in vertical direction. We know that for the static balance of the pulleys, the centre of gravity of the system must lie on the axis of rotation. Therefore a force polygon must be a closed figure. Now in Fig. 21.13 (*b*), draw *OA* parallel to vector *bc* and *OC* parallel to vector *co*. By measurement, we find that

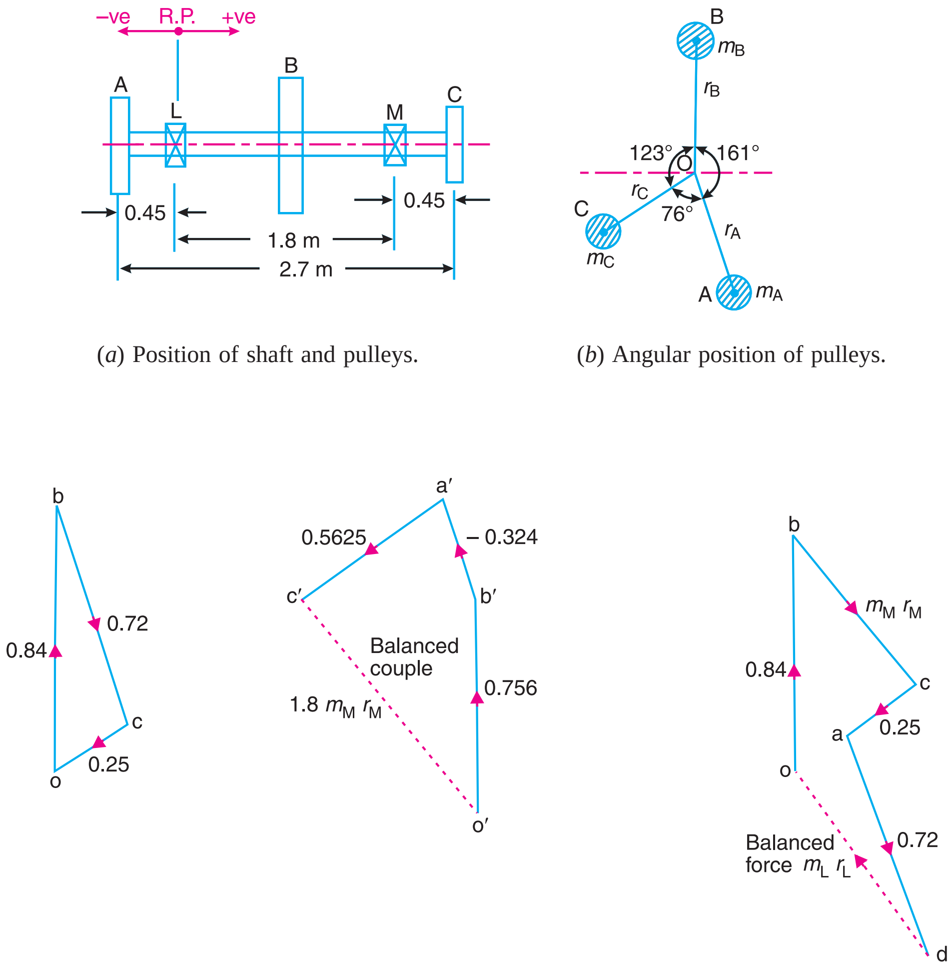
Angle between pulleys *B* and *A* = 161° **Ans.**

Angle between pulleys *A* and *C* = 76° **Ans.**

and Angle between pulleys *C* and *B* = 123° **Ans.**

2. *Dynamic forces at the two bearings*

In order to find the dynamic forces (or reactions) at the two bearings *L* and *M*, let us first calculate the values of $m_L.r_L$ and $m_M.r_M$ as discussed below :



(*a*) Position of shaft and pulleys. (*b*) Angular position of pulleys.

(*c*) Force polygon. (*d*) Couple polygon. (*e*) Force polygon.

Fig. 21.13

1. Draw the couple polygon to some suitable scale, as shown in Fig. 21.13 (*d*), from the data given in Table 21.7 (column 6). The closing side of the polygon (vector $c'o'$) represents the balanced couple and is proportional to 1.8 $m_M.r_M$. By measurement, we find that

$$1.8\ m_M.r_M = \text{vector } c'o' = 0.97 \text{ kg-m}^2 \quad \text{or} \quad m_M.r_M = 0.54 \text{ kg-m}$$

∴ Dynamic force at the bearing *M*

$$= m_M.r_M.\omega^2 = 0.54\ (31.42)^2 = 533 \text{ N} \ \textbf{Ans.}$$

2. Now draw the force polygon, as shown in Fig. 21.13 (*e*), from the data given in Table 21.7 (column 4) and taking $m_M.r_M$ = 0.54 kg-m. The closing side of the polygon (vector *do*) represents the balanced force and is proportional to $m_L.r_L$. By measurement, we find that

$$m_L.r_L = 0.54 \text{ kg-m}$$

∴ Dynamic force at the bearing *L*

$$= m_L.r_L.\omega^2 = 0.54\ (31.42)^2 = 533 \text{ N} \ \textbf{Ans.}$$

Notes : 1. The dynamic force at the two bearings are equal in magnitude but opposite in direction.

2. The dynamic force at the two bearings may also be obtained as discussed below :

From couple polygon as shown in Fig. 21.13 (*d*), we see that the vector $o'c'$ in the direction from o' to c' represents the out-of-balance couple.

By measurement, we find that

Out-of-balance couple

$$= \text{vector } o'c' = 0.97 \text{ kg-m}^2$$

$$= 0.97 \times \omega^2 = 0.97 \ (31.42)^2 = 957.6 \text{ N-m}$$

Since the shaft is in static balance, therefore it is only subjected to an unbalanced couple which is same about all planes and the bearing reactions are then equal and opposite. We know that

Dynamic force on each bearing

$$= \frac{\text{Out-of-balance couple}}{\text{Distance between bearings}} = \frac{957.6}{1.8} = 532 \text{ N} \quad \textbf{Ans.}$$

A spiral elevator conveyor for material handling.

Note : This picture is given as additional information and is not a direct example of the current chapter.

EXERCISES

1. Four masses *A*, *B*, *C* and *D* are attached to a shaft and revolve in the same plane. The masses are 12 kg, 10 kg, 18 kg and 15 kg respectively and their radii of rotations are 40 mm, 50 mm, 60 mm and 30 mm. The angular position of the masses *B*, *C* and *D* are 60°, 135° and 270° from the mass *A*. Find the magnitude and position of the balancing mass at a radius of 100 mm.

[Ans. 7.56 kg ; 87° clockwise from *A*]

2. Four masses *A*, *B*, *C* and *D* revolve at equal radii and are equally spaced along a shaft. The mass *B* is 7 kg and the radii of *C* and *D* make angles of 90° and 240° respectively with the radius of *B*. Find the magnitude of the masses *A*, *C* and *D* and the angular position of *A* so that the system may be completely balanced.

[Ans. 5 kg ; 6 kg ; 4.67 kg ; 205° from mass *B* in anticlockwise direction]

3. A rotating shaft carries four masses *A*, *B*, *C* and *D* which are radially attached to it. The mass centres are 30 mm, 38 mm, 40 mm and 35 mm respectively from the axis of rotation. The masses *A*, *C* and *D* are 7.5 kg, 5 kg and 4 kg respectively. The axial distances between the planes of rotation of *A* and *B* is 400 mm and between *B* and *C* is 500 mm. The masses *A* and *C* are at right angles to each other. Find for a complete balance,

1. the angles between the masses *B* and *D* from mass *A*,
2. the axial distance between the planes of rotation of *C* and *D*,
3. the magnitude of mass *B*. **[Ans. 162.5°, 47.5° ; 511 mm : 9.24 kg]**

4. A rotating shaft carries four unbalanced masses 18 kg, 14 kg, 16 kg and 12 kg at radii 50 mm, 60 mm, 70 mm and 60 mm respectively. The 2nd, 3rd and 4th masses revolve in planes 80 mm, 160 mm and 280 mm respectively measured from the plane of the first mass and are angularly located at 60°, 135° and 270° respectively measured clockwise from the first mass looking from this mass end of the shaft. The shaft is dynamically balanced by two masses, both located at 50 mm radii and revolving in planes mid-way between those of 1st and 2nd masses and midway between those of 3rd and 4th masses. Determine, graphically or otherwise, the magnitudes of the masses and their respective angular positions.

[Ans. 13.3 kg and 10.4 kg at 25° and 275° from mass *A* in anticlockwise direction]

5. A shaft carries five masses *A*, *B*, *C*, *D* and *E* which revolve at the same radius in planes which are equidistant from one another. The magnitude of the masses in planes *A*, *C* and *D* are 50 kg, 40 kg and 80 kg respectively. The angle between *A* and *C* is 90° and that between *C* and *D* is 135°. Determine the magnitude of the masses in planes *B* and *E* and their positions to put the shaft in complete rotating balance.

[Ans. 12 kg, 15 kg ; 130° and 24° from mass *A* in anticlockwise direction]

6. A shaft with 3 metres span between two bearings carries two masses of 10 kg and 20 kg acting at the extremities of the arms 0.45 m and 0.6 m long respectively. The planes in which these masses rotate are 1.2 m and 2.4 m respectively from the left end bearing supporting the shaft. The angle between the arms is 60°. The speed of rotation of the shaft is 200 r.p.m. If the masses are balanced by two counter-masses rotating with the shaft acting at radii of 0.3 m and placed at 0.3 m from each bearing centres, estimate the magnitude of the two balance masses and their orientation with respect to the *X*-axis, *i.e.* mass of 10 kg.

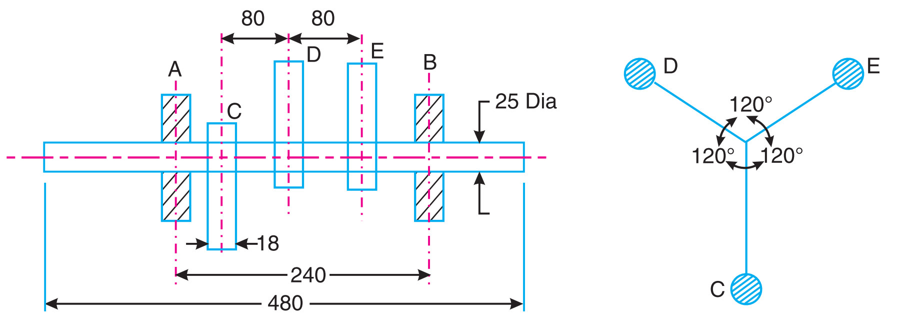
[Ans. 10 kg and 41 kg at 190° and 235° from *X*-axis in the anticlockwise direction]

7. *A*, *B*, *C* and *D* are four masses carried by a rotating shaft at radii 100 mm, 150 mm, 150 mm and 200 mm respectively. The planes in which the masses rotate are spaced at 500 mm apart and the magnitude of the masses *B*, *C* and *D* are 9 kg, 5 kg and 4 kg respectively. Find the required mass *A* and the relative angular settings of the four masses so that the shaft shall be in complete balance.

[Ans. 10 kg ; Between *B* and *A* 165°, Between *B* and *C* 295°, Between *B* and *D* 145°]

8. A 3.6 m long shaft carries three pulleys, two at its two ends and third at the mid-point. The two end pulleys has mass of 79 kg and 40 kg and their centre of gravity are 3 mm and 5 mm respectively from the axis of the shaft. The middle pulley mass is 50 kg and its centre of gravity is 8 mm from the shaft axis. The pulleys are so keyed to the shaft that the assembly is in static balance. The shaft rotates at 300 r.p.m. in two bearings 2.4m apart with equal overhang on either side. Determine : 1. the relative angular positions of the pulleys, and 2. dynamic reactions at the two bearings.

9. The camshaft of high speed pump consists of a parallel shaft 25 mm diameter and 480 mm long. It carries three eccentrics, each of diameter 60 mm and a uniform thickness of 18 mm. The assembly is symmetrical as shown in Fig. 21.14 and the bearings are at *A* and *B*. The angle between the eccentrics is 120° and the eccentricity of each is 12.5 mm. The material density is 7000 kg/m^3, and the speed of rotation is 1430 r.p.m.



All dimensions in mm.

Fig. 21.14

Find : **1.** dynamic load on each bearing due to the out-of-balance couple ; and **2.** kinetic energy of the complete assembly. **[Ans. 6.12 kg ; 8.7 N-m]**

DO YOU KNOW ?

1. Why is balancing of rotating parts necessary for high speed engines ?

2. Explain clearly the terms 'static balancing' and 'dynamic balancing'. State the necessary conditions to achieve them.

3. Discuss how a single revolving mass is balanced by two masses revolving in different planes.

4. Explain the method of balancing of different masses revolving in the same plane.
5. How the different masses rotating in different planes are balanced ?

OBJECTIVE TYPE QUESTIONS

1. The balancing of rotating and reciprocating parts of an engine is necessary when it runs at
   (*a*) slow speed (*b*) medium speed (*c*) high speed
2. A disturbing mass m_1 attached to a rotating shaft may be balanced by a single mass m_2 attached in the same plane of rotation as that of m_1 such that
   (*a*) $m_1.r_2 = m_2.r_1$ (*b*) $m_1.r_1 = m_2.r_2$ (*c*) $m_1.\ m_2 = r_1.r_2$
3. For static balancing of a shaft,
   (*a*) the net dynamic force acting on the shaft is equal to zero
   (*b*) the net couple due to the dynamic forces acting on the shaft is equal to zero
   (*c*) both (*a*) and (*b*)
   (*d*) none of the above
4. For dynamic balancing of a shaft,
   (*a*) the net dynamic force acting on the shaft is equal to zero
   (*b*) the net couple due to dynamic forces acting on the shaft is equal to zero
   (*c*) both (*a*) and (*b*)
   (*d*) none of the above
5. In order to have a complete balance of the several revolving masses in different planes
   (*a*) the resultant force must be zero
   (*b*) the resultant couple must be zero
   (*c*) both the resultant force and couple must be zero
   (*d*) none of the above

ANSWERS

1. (*c*) **2.** (*b*) **3.** (*a*) **4.** (*c*) **5.** (*c*)

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